Extended HQEFT Lagrangian and currents *

F. Bertó[a] and M. A. Sanchis-Lozano[a,b†]

(a) Departamento de Física Teórica

(b) Instituto de Física Corpuscular (IFIC)
Centro Mixto Universitat de València-CSIC
Dr. Moliner 50, E-46100 Burjassot, Valencia (Spain)

From the tree-level heavy quark effective Lagrangian keeping particle-antiparticle mixed sectors we derive the vector current coupling to a hard gluonic field allowing for heavy quark-antiquark pair annihilation and creation.

1. A COMPLETE TREE-LEVEL HQEFT LAGRANGIAN

In this work we keep both heavy quark and heavy antiquark coupled sectors in the HQEFT Lagrangian. To our knowledge, until [1] only the paper by Wu [2] in the literature actually dealt with both heavy quark and antiquark fields altogether in the effective Lagrangian. In the present work, based on [3,1], we pursue this line of investigation further, looking for more symmetric expressions and a more transparent physical interpretation.

Following the standard reference [4] we introduce the effective fields for a heavy quark bound inside a hadron moving with (four-)velocity v, as

$$h_v^{(+)}(x) \;=\; e^{imv\cdot x}\,\frac{1+\not{v}}{2}\,Q^{(+)}(x) \tag{1}$$

$$H_v^{(+)}(x) \;=\; e^{imv\cdot x}\,\frac{1-\not{v}}{2}\,Q^{(+)}(x) \tag{2}$$

where $Q^{(+)}(x)$ stands for the (positive energy) fermionic field describing the heavy quark in the full theory; $h_v^{(+)}(x)$ and $H_v^{(+)}(x)$ represent the "large" and "small" components of a classical spinor field respectively.

Similarly for a heavy antiquark,

$$h_v^{(-)}(x) \;=\; e^{-imv\cdot x}\,\frac{1-\not{v}}{2}\,Q^{(-)}(x) \tag{3}$$

$$H_v^{(-)}(x) \;=\; e^{-imv\cdot x}\,\frac{1+\not{v}}{2}\,Q^{(-)}(x) \tag{4}$$

where $Q^{(-)}(x)$ stands for the (negative energy) anti-fermionic field in the full theory.

Introducing the residual momentum q for a heavy quark of total momentum p as $p = mv + q$, the on-shell condition can be written as

$$v\cdot q = -\frac{q^2}{2m} \tag{5}$$

The underlying idea when introducing the residual momentum is that once removed the large mechanical momentum associated to the heavy-quark mass, only the low-energy modes remain in the effective theory [5]. This is the standard way to handle heavy quarks in singly heavy hadrons according to HQET. However, one may conjecture about the possibility of performing such an energy-momentum shift by introducing a center-of-mass residual momentum for processes involving creation or annihilation of heavy quark-antiquark pairs at tree-level. Hence only low energy modes of the fields (about the heavy quark mass) would be involved likely making meaningful our approach within the framework of an effective theory.

In sum, the main difference of this paper w.r.t. other standard works is that we are concerned with the existence of terms in the transformed Lagrangian mixing the large components of the heavy quark and heavy antiquark fields, i.e. $h_v^{(\pm)}\Gamma h_v^{(\mp)}$, where Γ stands for a combination of Dirac gamma matrices and covariant derivatives,

*Work partially supported by CICYT under contract AEN99-0692
†E-mail: mas@evalo1.ific.uv.es

instead of directly writing currents as bilinears mixing both particle and antiparticle sectors as in [6,7].

The tree-level QCD Lagrangian is our point of departure:

$$\mathcal{L} \ = \ \overline{Q}\ (i\overrightarrow{\not{D}} - m)\ Q \qquad (6)$$

where

$$Q \ = \ Q^{(+)} + Q^{(-)} \ = \qquad (7)$$
$$e^{-imv\cdot x}\left[h_v^{(+)} + H_v^{(+)}\right] + e^{imv\cdot x}\left[h_v^{(-)} + H_v^{(-)}\right]$$

and D standing for the covariant derivative

$$\overrightarrow{D}^{\mu} \ = \ \overrightarrow{\partial}^{\mu} \ - \ igT_aA_a^{\mu}$$

with T_a the generators of $SU(3)_c$. Substituting (7) in (8) one easily arrives at

$$\mathcal{L} \ = \ \mathcal{L}^{(++)} \ + \ \mathcal{L}^{(--)} \ + \ \mathcal{L}^{(-+)} \ + \ \mathcal{L}^{(+-)} \qquad (8)$$

where we have explicitly splitted the Lagrangian into four different pieces corresponding to the particle-particle, antiparticle-antiparticle and both particle-antiparticle sectors. The former one has the form

$$\mathcal{L}^{(++)} = \overline{h}_v^{(+)} iv\cdot\overrightarrow{D}h_v^{(+)} - \overline{H}_v^{(+)}(iv\cdot\overrightarrow{D} + 2m)H_v^{(+)}$$
$$+\ \overline{h}_v^{(+)} i\overrightarrow{\not{D}}_{\perp}H_v^{(+)} + \overline{H}_v^{(+)} i\overrightarrow{\not{D}}_{\perp}h_v^{(+)}$$

corresponding to the usual HQET Lagrangian still containing both $h_v^{(+)}$ and $H_v^{(+)}$ fields. We employ the common notation where perpendicular indices are implied according to

$$D_{\perp}^{\mu} \ = \ D_{\alpha}\ (g^{\mu\alpha} - v^{\mu}v^{\alpha})$$

Regarding the antiquark sector of the theory

$$\mathcal{L}^{(--)} = -\overline{h}_v^{(-)} iv\cdot\overrightarrow{D}h_v^{(-)} + \overline{H}_v^{(-)}(iv\cdot\overrightarrow{D} - 2m)H_v^{(-)}$$
$$+\ \overline{h}_v^{(-)} i\overrightarrow{\not{D}}_{\perp}H_v^{(-)} + \overline{H}_v^{(-)} i\overrightarrow{\not{D}}_{\perp}h_v^{(-)}$$

The latter expressions, considered as quantum field Lagrangians , do not afford tree-level heavy quark-antiquark pair creation or annihilation processes stemming from the terms mixing $h_v^{(\pm)}$ and $H_v^{(\pm)}$ fields since they contain either annihilation and creation operators of heavy quarks or annihilation and creation operators of heavy antiquarks separately.

Nevertheless there are two extra pieces in the Lagrangian (8):

$$\mathcal{L}^{(-+)} = e^{-i2mv\cdot x}\times$$
$$[\ \overline{H}_v^{(-)} iv\cdot\overrightarrow{D}h_v^{(+)} - \overline{h}_v^{(-)}(iv\cdot\overrightarrow{D} + 2m)H_v^{(+)}$$
$$+\ \overline{h}_v^{(-)} i\overrightarrow{\not{D}}_{\perp}h_v^{(+)} + \overline{H}_v^{(-)} i\overrightarrow{\not{D}}_{\perp}H_v^{(+)}\ ]$$

and

$$\mathcal{L}^{(+-)} = e^{i2mv\cdot x}\ \times$$
$$[\ -\overline{H}_v^{(+)} iv\cdot\overrightarrow{D}h_v^{(-)} + \overline{h}_v^{(+)}(iv\cdot\overrightarrow{D} - 2m)H_v^{(-)}$$
$$+\ \overline{h}_v^{(+)} i\overrightarrow{\not{D}}_{\perp}h_v^{(-)} + \overline{H}_v^{(+)} i\overrightarrow{\not{D}}_{\perp}H_v^{(-)}\ ]$$

where use was made of the orthogonality of the $h_v^{(\pm)}$ and $H_v^{(\pm)}$ fields. As could be expected, there are indeed pieces mixing both quark and antiquark fields leading to the possibility of annihilation/creation processes deriving directly from the HQET Lagrangian. After all the Lagrangian (8) is still equivalent to full (tree-level) QCD [3].

Let us note that, at first sight, one might think that the rapidly oscillating exponential would make both $\mathcal{L}^{(-+)}$ and $\mathcal{L}^{(+-)}$ pieces to vanish, once integrated over all velocities according to the most general Lagrangian [8]. However, notice that actually this should not be the case for momenta of order $2mv$ of the gluonic field present in the covariant derivative. In fact, only such high-energy modes will survive, corresponding to the physical situation on which we are focusing, i.e. heavy quark-antiquark pair annihilation and creation.

The heavy quark-gluon coupling for an annihilation process can be read off from the Lagrangian

[3] Superscript "(+)/(−)" on the effective fields labels the particle/antiparticle sector of the theory [8]. Actually $\overline{h}_v^{(+)}$ ($\overline{h}_v^{(-)}$) corresponds to negative (positive) frequencies associated with creation (annihilation) operators of quarks (antiquarks). In fact some extra "$+/-$" signs should be added on the conjugate fields, i.e. $\overline{h}_v^{(+)-}$ and $\overline{h}_v^{(-)+}$, which however will be omitted to shorten the notation.

piece:

$$\begin{aligned}\mathcal{L}^{(-+)}_{coupling} &= e^{-i2mv\cdot x}\ gT_a A^a_\mu\ \times \\ & [\quad \overline{H}^{(-)}_v\ v^\mu\ h^{(+)}_v \qquad (a) \\ & -\quad \overline{h}^{(-)}_v\ v^\mu\ H^{(+)}_v \qquad (b) \\ & +\quad \overline{h}^{(-)}_v\ \gamma^\mu_\perp\ h^{(+)}_v \qquad (c) \\ & +\quad \overline{H}^{(-)}_v\ \gamma^\mu_\perp\ H^{(+)}_v\ ] \quad (d)\end{aligned} \tag{9}$$

Next we want to eliminate the unwanted degrees of freedom associated to the "small" components $H^{(\pm)}_v$ in (a), (b) and (d). The piece (c) is the leading one in the above development.

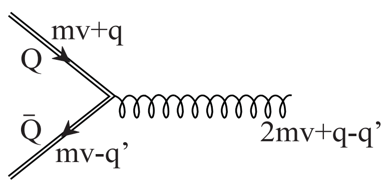

Figure 1. Heavy quark-antiquark pair annihilation into a gluon of momentum $2mv + q - q'$. In the center-of mass system $q = (q_0, \mathbf{q})$ and $q' = (-q_0, \mathbf{q})$. This process can be described meaningfully by HQEFT if $\mathbf{q^2} < m^2$, i.e. the square invariant mass of the gluon is close to $4m^2$.

2. DERIVATION OF THE ANNIHILATION VERTEX

Assuming the same conditions as in figure 1, we can write for almost free heavy quarks in the initial- or final-state:

$$H^{(+)}_v = (iv\cdot\overrightarrow{\partial} + 2m - i\epsilon)^{-1} i\overrightarrow{\partial\!\!\!/}_\perp h^{(+)}_v \tag{10}$$

$$H^{(-)}_v = (-iv\cdot\overrightarrow{\partial} + 2m - i\epsilon)^{-1} i\overrightarrow{\partial\!\!\!/}_\perp h^{(-)}_v \tag{11}$$

and for the conjugate fields

$$\overline{H}^{(+)}_v = \overline{h}^{(+)}_v i\overleftarrow{\partial\!\!\!/}_\perp (iv\cdot\overleftarrow{\partial} - 2m + i\epsilon)^{-1} \tag{12}$$

$$\overline{H}^{(-)}_v = \overline{h}^{(-)}_v i\overleftarrow{\partial\!\!\!/}_\perp (-iv\cdot\overleftarrow{\partial} - 2m + i\epsilon)^{-1} \tag{13}$$

Let us note that we are using the free particle equations which can be viewed as derived from the non-interacting parts of the Lagrangians $\mathcal{L}^{(++)}$ and $\mathcal{L}^{(--)}$ respectively. In fact, neglecting the soft gluon interaction among heavy quarks amounts to describe them as plane waves, i.e. actually no bound states as a first approximation [9]. Therefore assuming a plane wave dependence of the field quantities, we can write in momentum space

$$u(mv + q) = \left[\ 1 + \frac{q\!\!\!/_\perp}{2m + v\cdot q}\ \right] u_h(q) \tag{14}$$

$$\overline{v}(mv - q') = \overline{v}_h(-q') \left[\ 1 - \frac{q\!\!\!/'_\perp}{2m - v\cdot q'}\ \right] \tag{15}$$

where $u(p)$ $(v(p'))$ denotes the full QCD spinor (antispinor) whereas $u_h(q)$ $(v_h(-q'))$ represents the HQET spinor (antispinor), i.e. obeying $v\!\!\!/ u_h = u_h$ $(v\!\!\!/ v_h = -v_h)$.

Actually, in order to arrive to Eqs. (14-15) from Eqs. (10-13) one has to expand the denominators as power series of derivatives acting on the x-dependent factors, assumed exponentials, to be finally resummed as a geometric series of ratio $v\cdot q/2m = -v\cdot q'/2m = -q^2/4m^2$ according to the on-shell condition (5). As a consequence, Eqs. (14-15) are only valid under the condition $-q^2 < 4m^2$, which implies $\mathbf{q^2} < 8m^2$. Therefore, the requirement $\mathbf{q^2} < m^2$ satisfies the above condition and will allow a later non-relativistic expansion.

Substituting the above equations into the $\mathcal{L}^{(-+)}_{coupling}$ Lagrangian (9), we readily get for the on-shell heavy quark (vector) current coupling to a gluon, suppressing color indices and matrices

$$\overline{v}_h \left[\gamma^\mu_\perp + \frac{q\!\!\!/'_\perp - q\!\!\!/_\perp}{2m + v\cdot q} v^\mu + \frac{q\!\!\!/'_\perp \gamma^\mu q\!\!\!/_\perp}{(2m + v\cdot q)^2}\right] u_h$$

which can also be written as

$$\overline{v}_h \left[\gamma^\mu_\perp + \frac{i\sigma^{\mu\nu}(q'_\perp - q_\perp)_\nu}{2m + v\cdot q} + \frac{q\!\!\!/'_\perp \gamma^\mu q\!\!\!/_\perp}{(2m + v\cdot q)^2}\right] u_h \tag{16}$$

since [1]

$$P_-\ (v^\mu \gamma^\nu)\ P_+\ =\ P_-\ (i\sigma^{\mu\nu}\ +\ \gamma^\mu v^\nu)\ P_+$$

with the projectors $P_\pm = (1 \pm v\!\!\!/)/2$, and $q'_{\perp\nu} v^\nu = q_{\perp\nu} v^\nu = 0$.

2.1. CENTER-OF-MASS FRAME

We shall make use of the anticommutation relation:

$$\not{q}'_{\perp}\gamma^{\mu}\not{q}_{\perp} = 2q'^{\mu}_{\perp}\not{q}_{\perp} - \gamma^{\mu}\not{q}'_{\perp}\not{q}_{\perp}$$

Now, in the center-of-mass frame we can write $\not{q}'_{\perp}\not{q}_{\perp} = -\mathbf{q^2}$, leading to the expression for the heavy quark vector current

$$\overline{v}_h \left[\frac{2E_q}{E_q + m}\gamma^{\mu}_{\perp} + \frac{2q'^{\mu}_{\perp}\not{q}_{\perp}}{(E_q + m)^2} \right] u_h \tag{17}$$

where $E_q = m + q^0 = \sqrt{m^2 + \mathbf{q}^2}$. The following identity is satisfied (in the Dirac representation):

$$P_- \ \not{q}_{\perp} \ P_+ = \begin{pmatrix} 0 & 0 \\ \sigma{\cdot}\mathbf{q} & 0 \end{pmatrix} \tag{18}$$

Therefore identifying as two component spinors $u_h = \sqrt{E_q + m} \ \xi$, and $v_h = \sqrt{E_q + m} \ \eta$, we obtain from (17) the analogous expression to Eq. (A.9b) of Ref. [7],

$$\eta^{\dagger} \left[2E_q \ \sigma + \frac{2\mathbf{q} \ \sigma{\cdot}\mathbf{q}}{(E_q + m)} \right] \xi \tag{19}$$

i.e. the vertex obtained from full QCD in terms of the Pauli spinors ξ and η, allowing a systematic non-relativistic expansion: the leading term $2m(\eta^{\dagger}\sigma\xi)$ associated with (c) in (9) as expected.

3. SUMMARY AND LAST REMARKS

We have derived a complete tree-level heavy-quark transformed Lagrangian in terms of the effective fields $h_v^{(\pm)}$, keeping the particle-antiparticle mixed pieces allowing for heavy quark-antiquark pair annihilation/creation. Let us note that such pieces are not generally shown in similar developments in the literature.

Indeed, it may seem quite striking that a low-energy effective theory could be appropriate to deal with hard processes such as $Q\overline{Q}$ annihilation or creation. The keypoint is that assuming a kinematic regime where heavy quarks/antiquarks are almost on-shell and moving with small relative velocity, the strong momentum dependence associated with the heavy quark masses can be factored out, so that a description based on the low frequency modes of the fields still makes sense. Such a kinematic regime is well matched by heavy quarkonium states and by colored intermediate states in the so-called color-octet model recently introduced [10] to account for high production rates of heavy quarkonia at the Fermilab Tevatron [11].

In particular, we have focused on an annihilation process with initial-state quarks satisfying the Dirac equation of motion for free fermions. Thus, we have derived directly from the $\mathcal{L}^{(-+)}$ piece the heavy quark vector current coupling to a background gluonic field, recovering a well known expression shown in the literature [7]. A similar development can be done for a heavy quark pair creation process from the $\mathcal{L}^{(+-)}$ piece.

The main point stressed in this paper is that one can derive the heavy quark/antiquark vector current coupling to a hard gluonic field from the corresponding piece of the transformed tree-level Lagrangian (8), without resorting to an *ad hoc* construction from the fermionic spinors themselves, perhaps providing a more self-consistent basis to the matching procedure of the effective theory onto full QCD [7].

REFERENCES

1. F. Bertó, J.L. Domenech and M.A. Sanchis-Lozano, Nuov. Cim. **A112** (1999) 1181 (hep-ph/9810549).
2. Y.L. Wu, Mod. Phys. Lett. **A8** (1993) 819.
3. M.A. Sanchis-Lozano, Nuov. Cim. **A110** (1997) 295 (hep-ph/9612210).
4. M. Neubert, Phys. Rep. **245** (1994) 259.
5. U. Aglietti and S. Capitani, Nucl. Phys. **B432** (1994) 315.
6. T. Mannel and G. Schuler, Z. Phys. **C67** (1995) 159.
7. E. Braaten and Y-Q Chen, Phys. Rev. **D55** (1997) 2693; **D54** (1996) 3216.
8. H. Georgi, Phys. Lett. **B240** (1990) 447.
9. F. Hussain, J.G. Körner and G. Thompson, Ann. Phys. **206** (1991) 334.
10. E. Braaten and S. Fleming, Phys. Rev. Lett. **74** (1995) 3327.
11. CDF Collaboration, Phys. Rev. Lett. **69** (1992) 3704; **79** (1997) 572.